FACULTAD DE INGENIERIA
Universidad de Buenos Aires

Guía 1: Electrostática en el vacío

Campo electrostático

9. Plantear la expresión para el cálculo del campo eléctrico, en todo punto del espacio, producido por una distribución esférica de carga de radio R y de densidad volumétrica $\rho = \rho_0 \cos\varphi$

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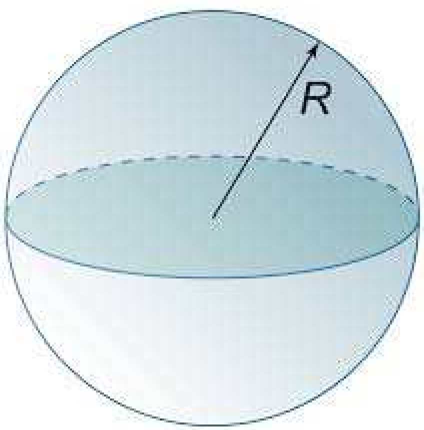

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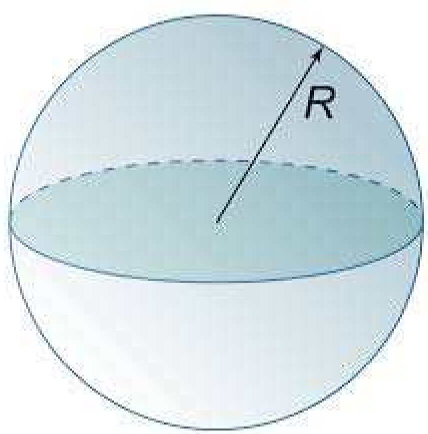
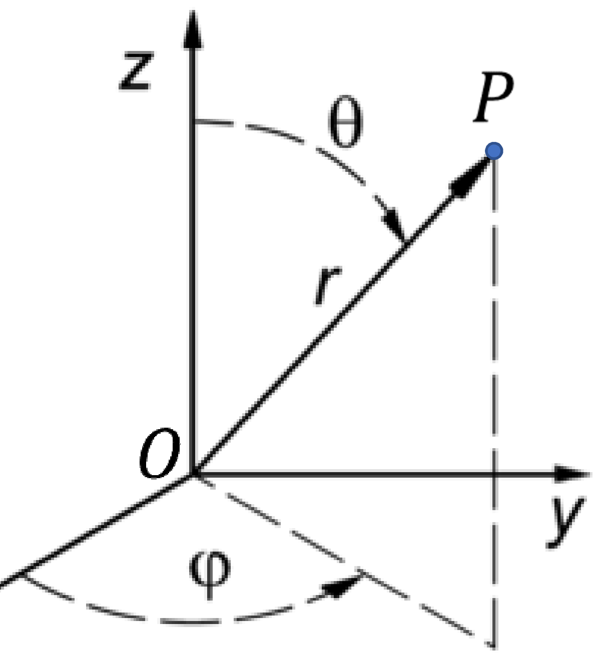


Coordenadas esféricas: r, θ, φ


z
θ
P
r
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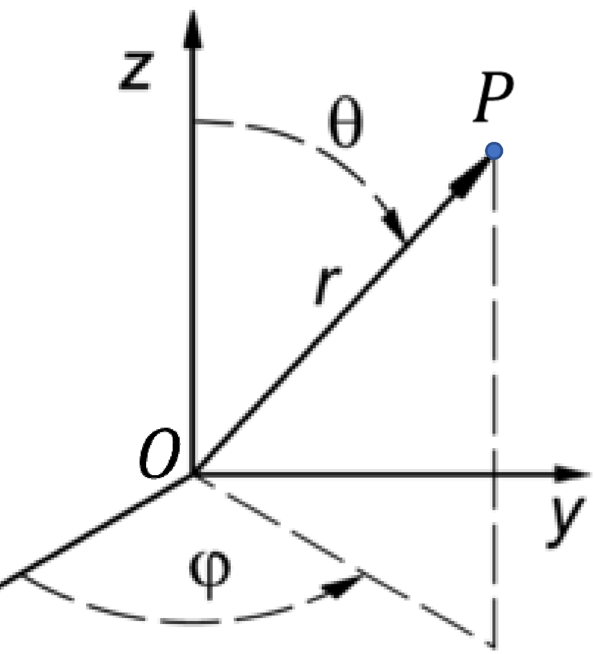

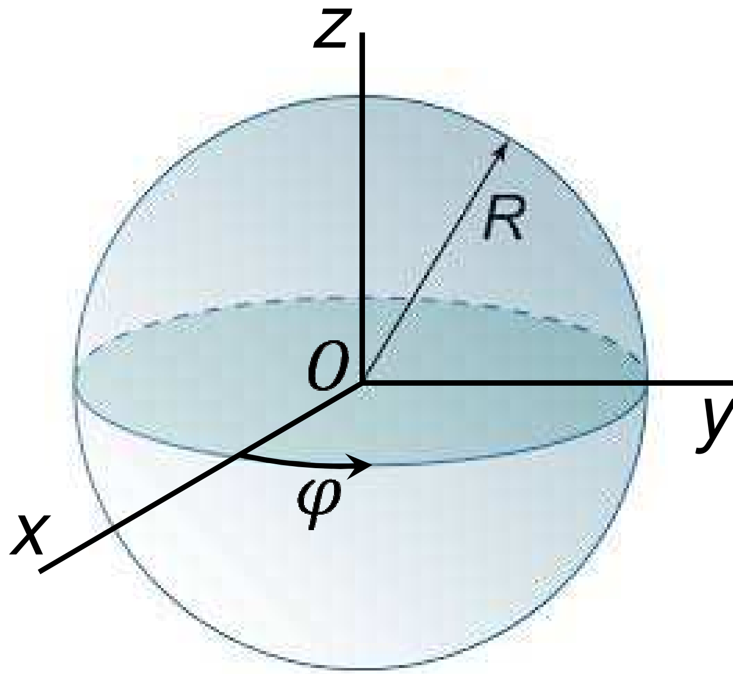

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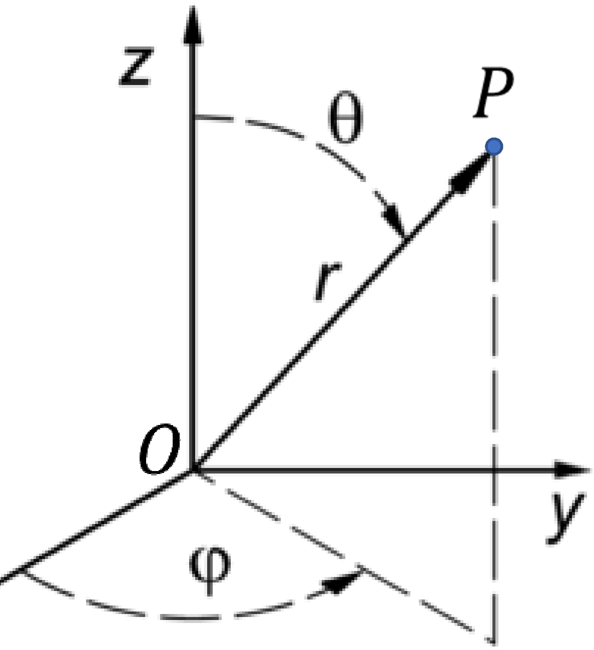

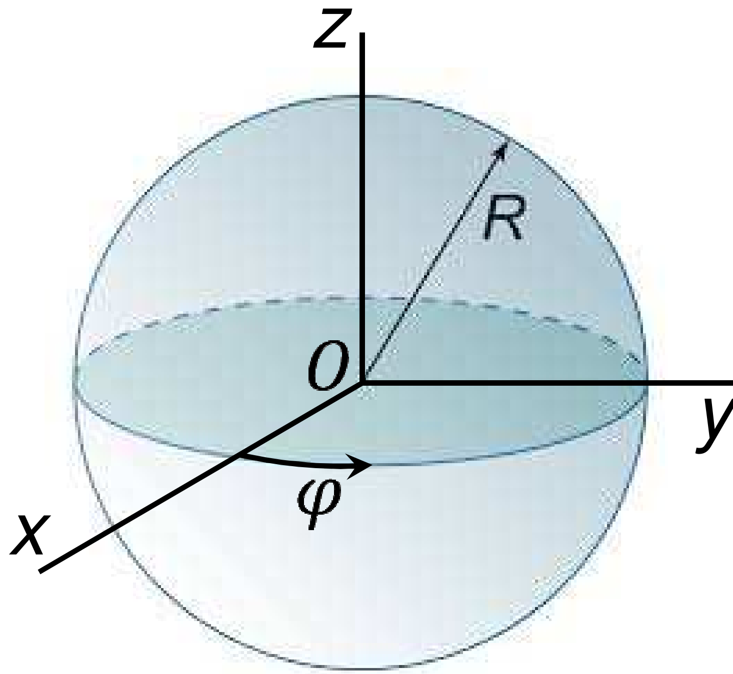

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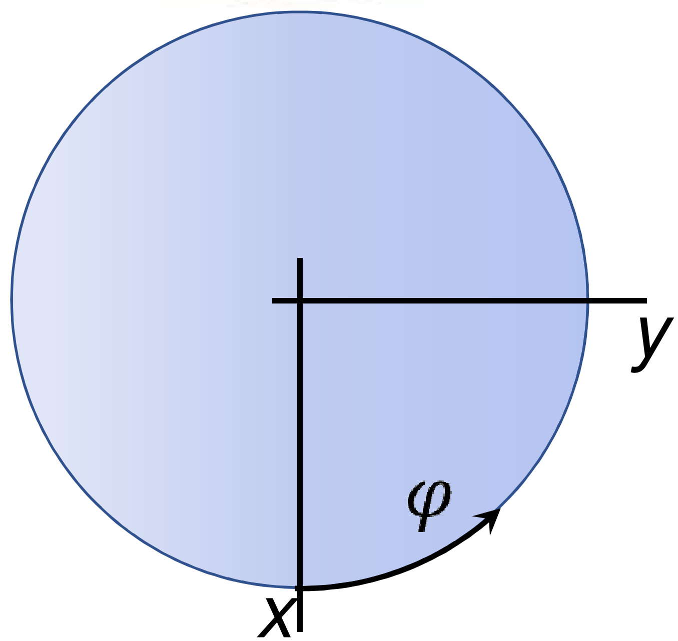

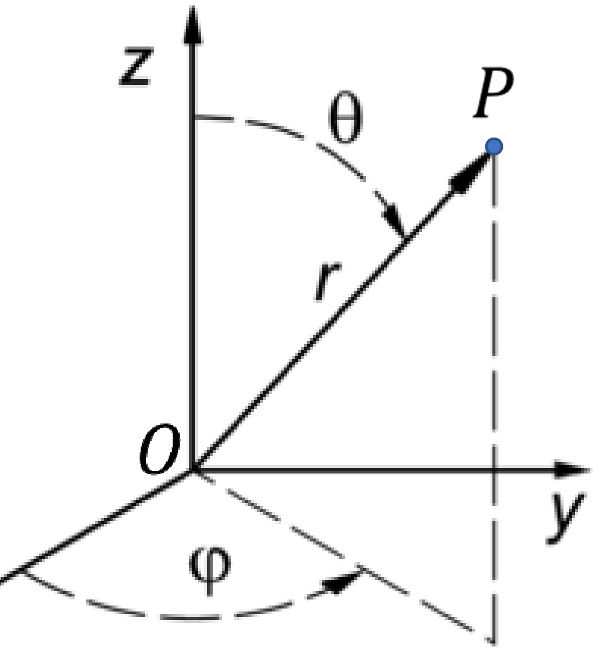

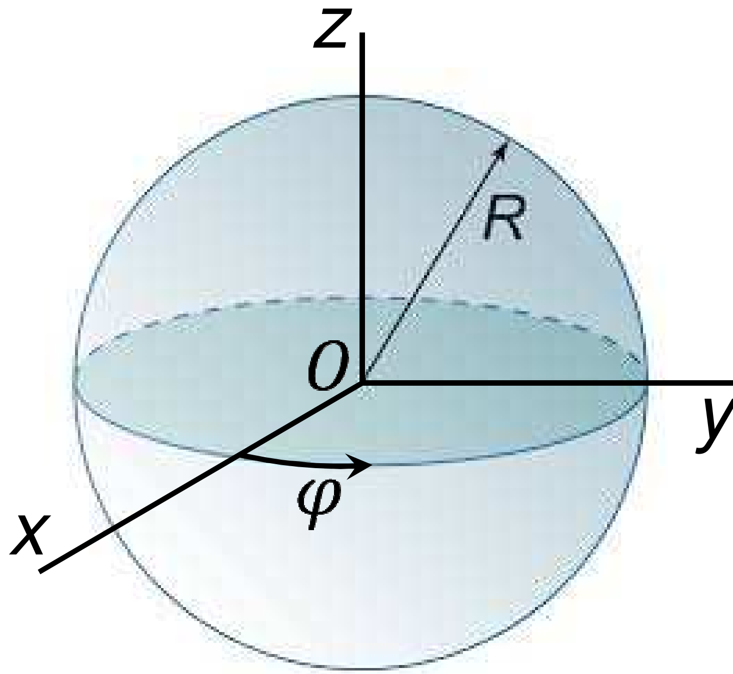

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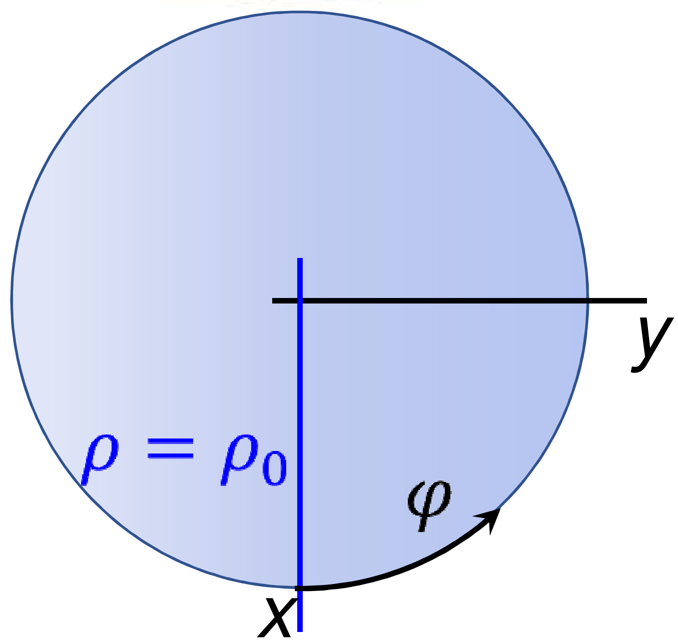

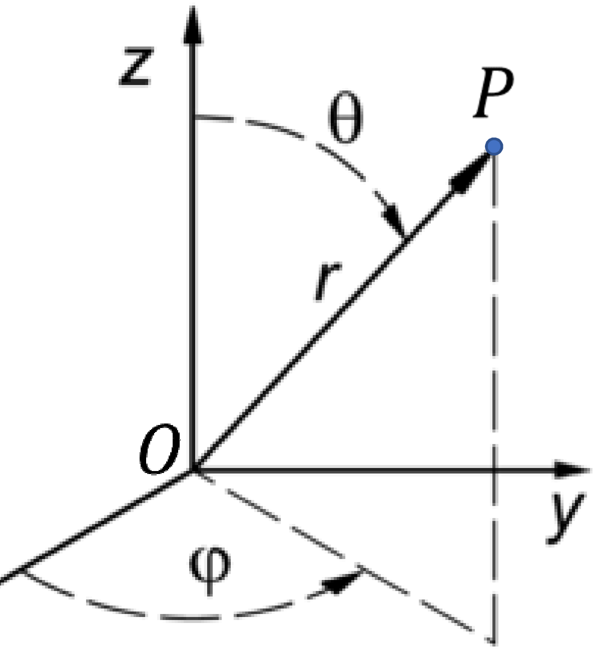

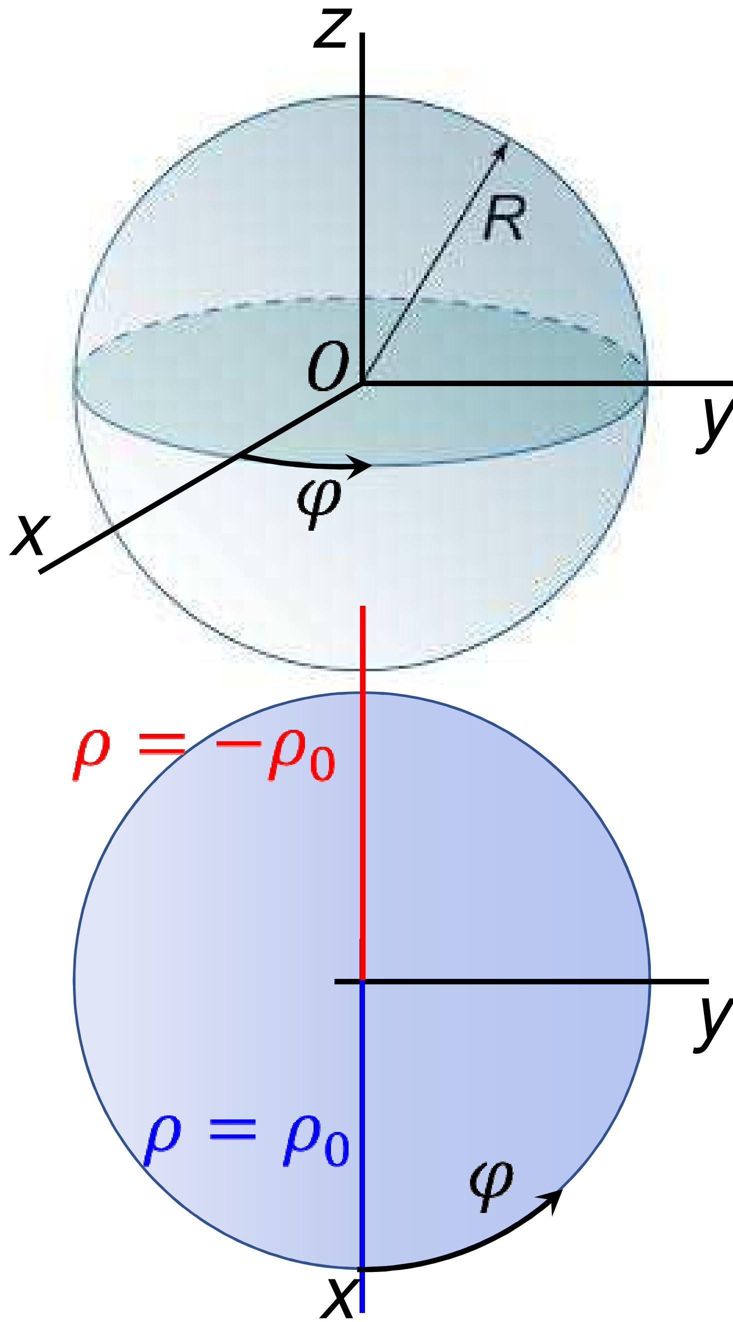

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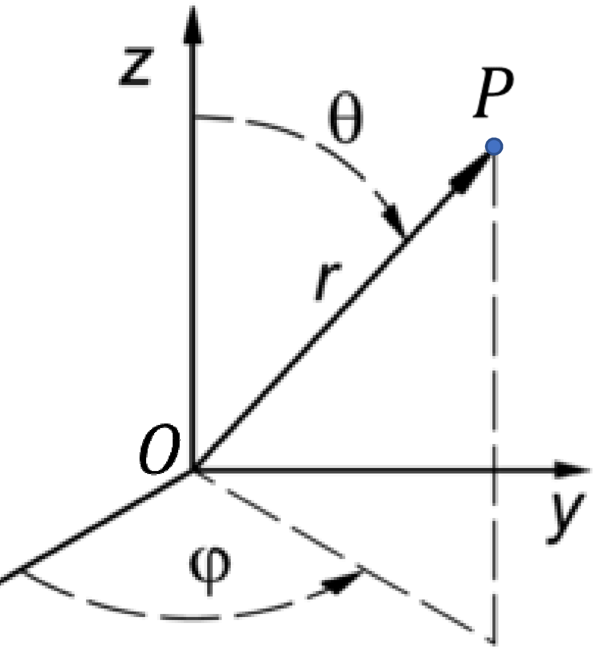

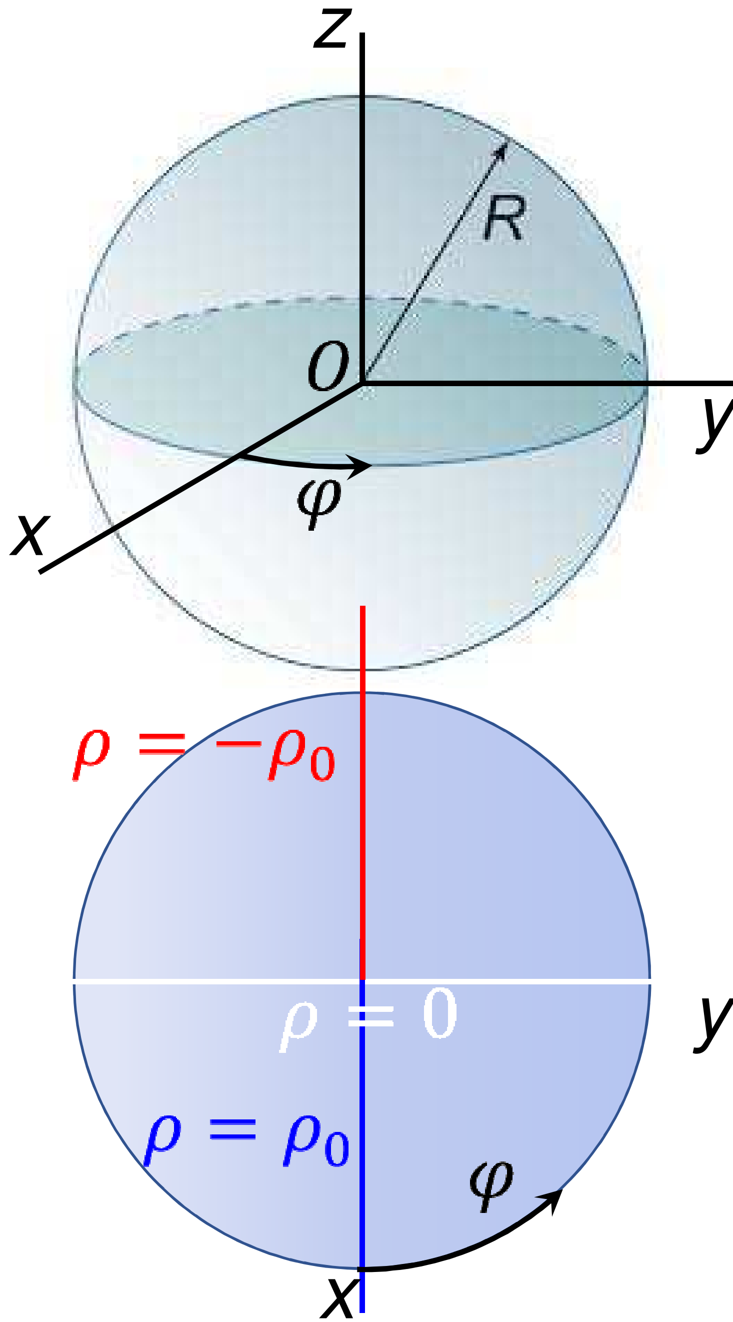

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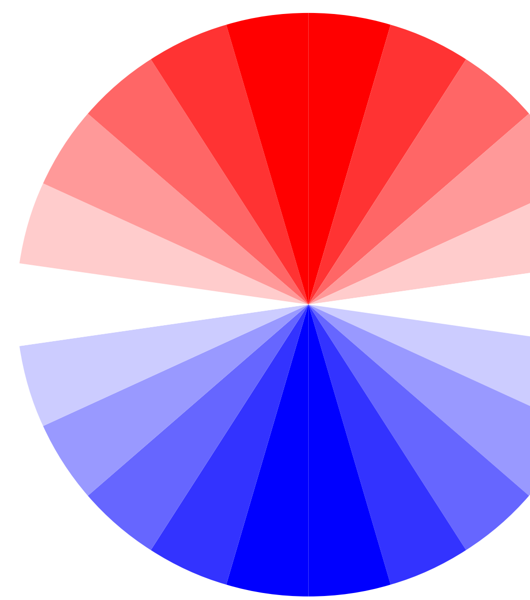

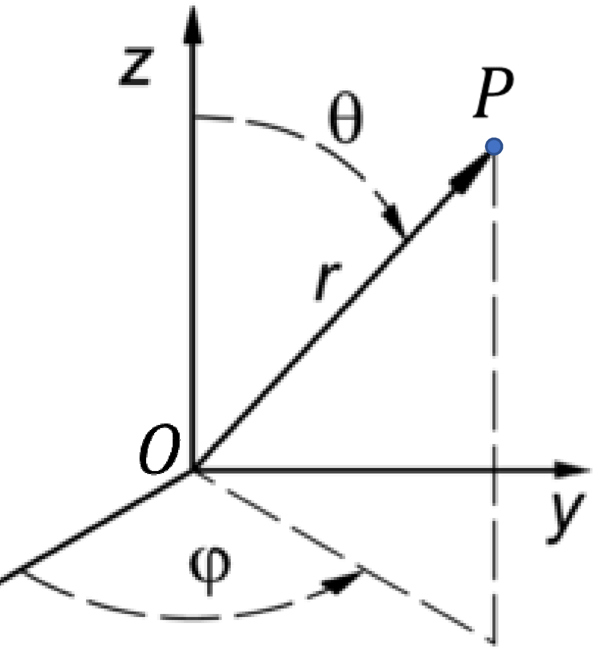

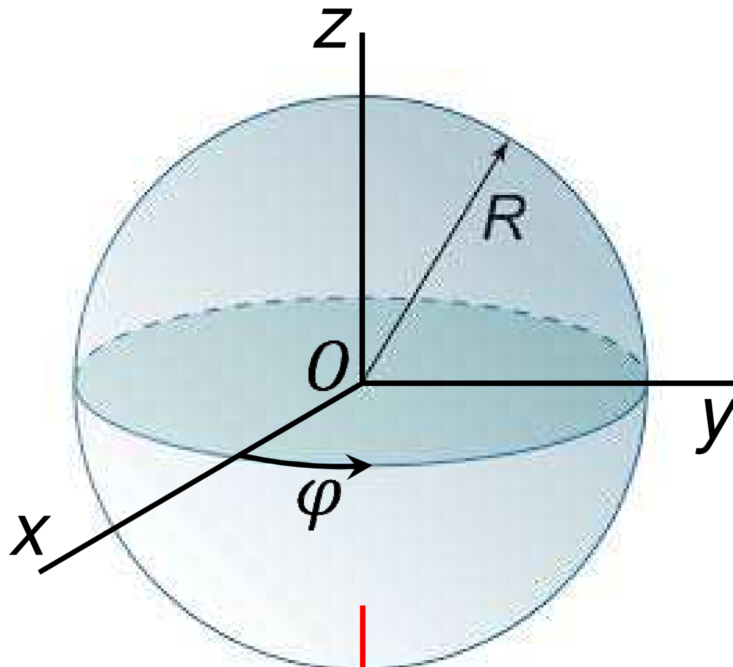

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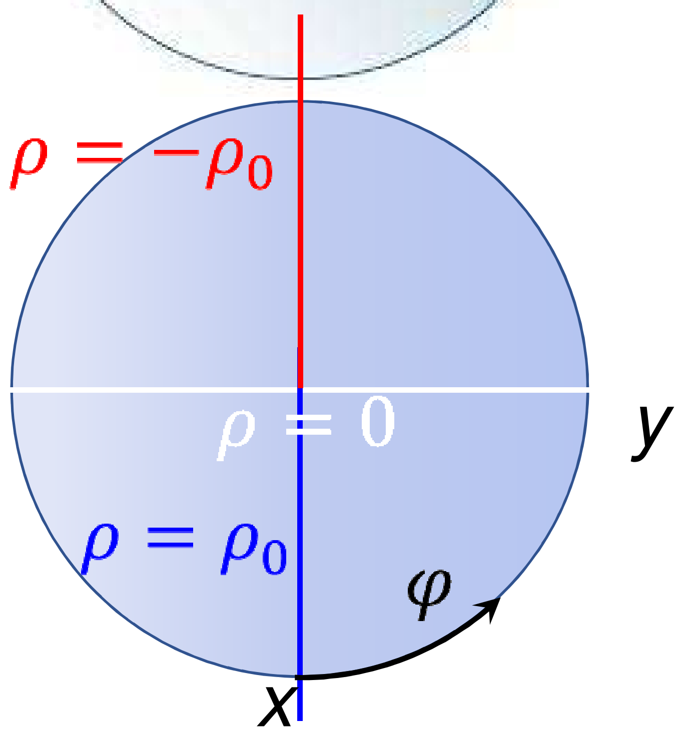
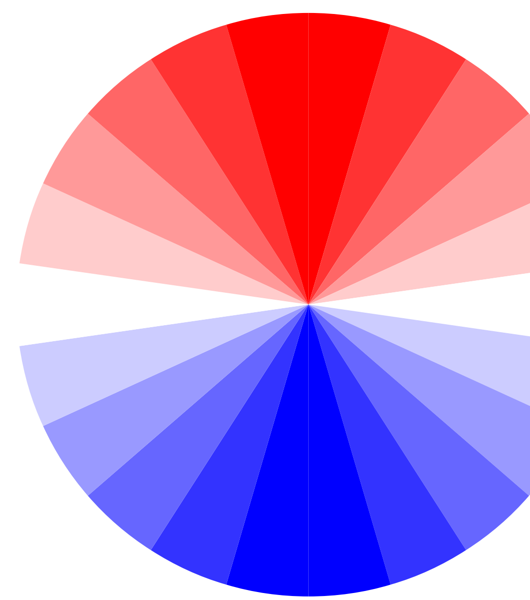
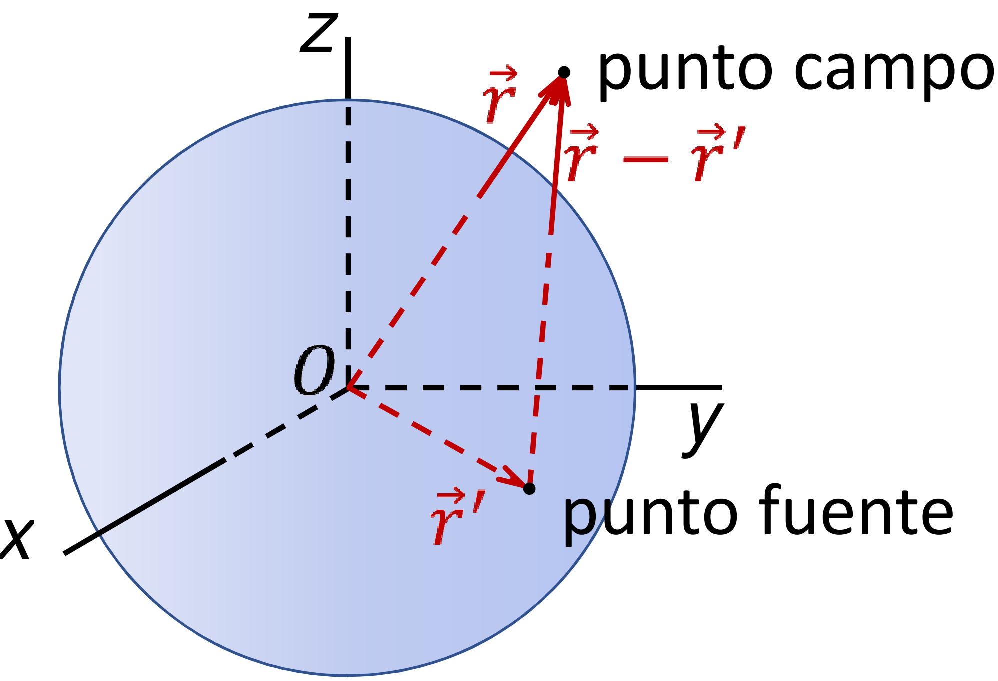




$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\mathrm{d}V'}{|\vec{r}-\vec{r}\,'|^3}$$



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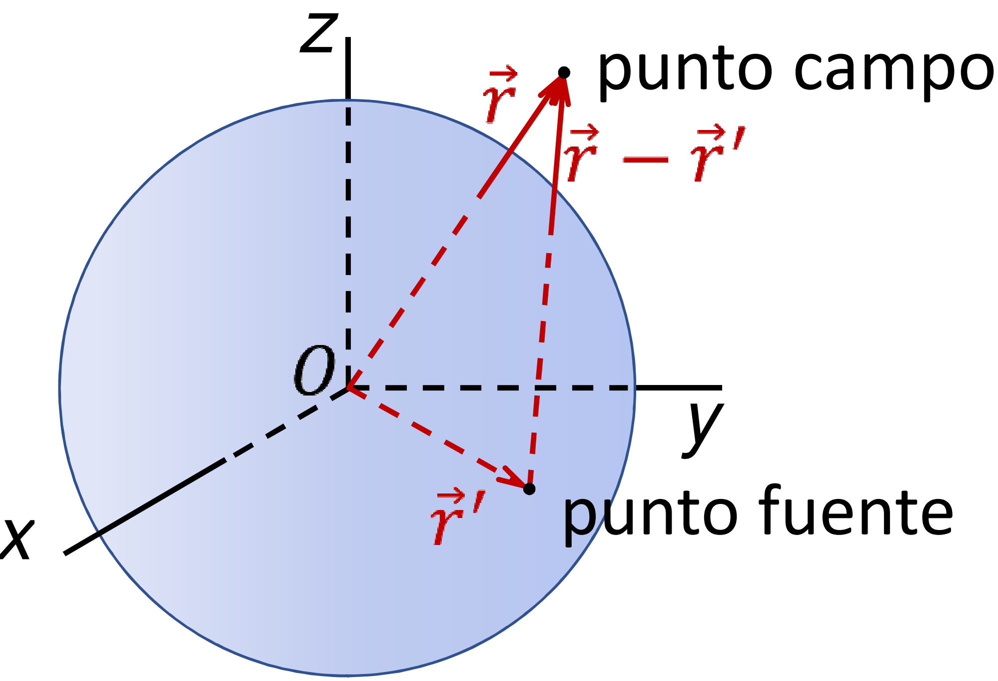

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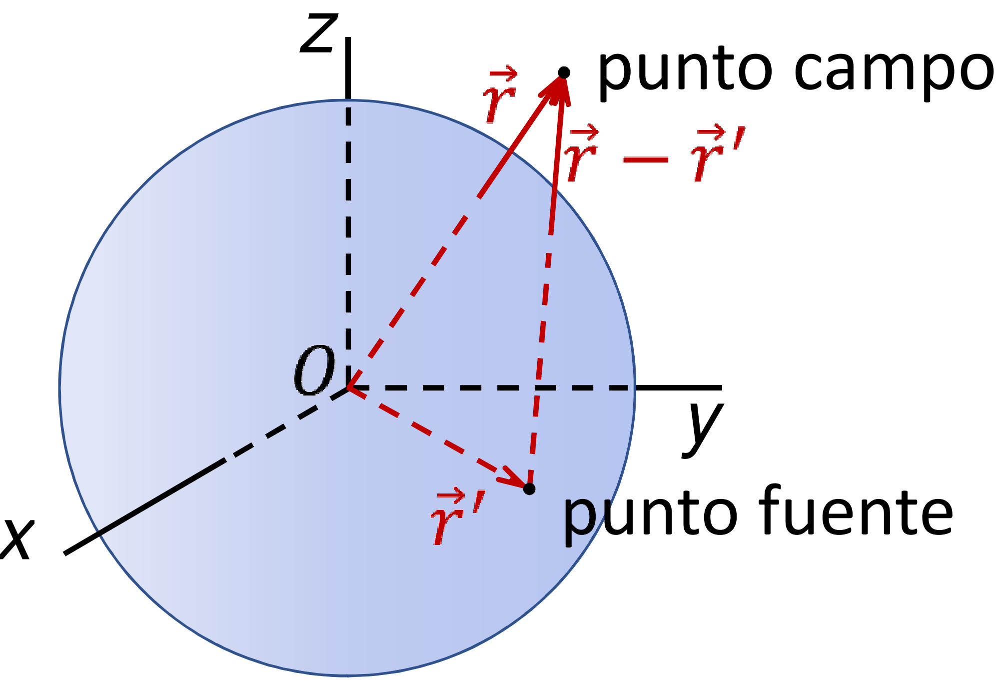

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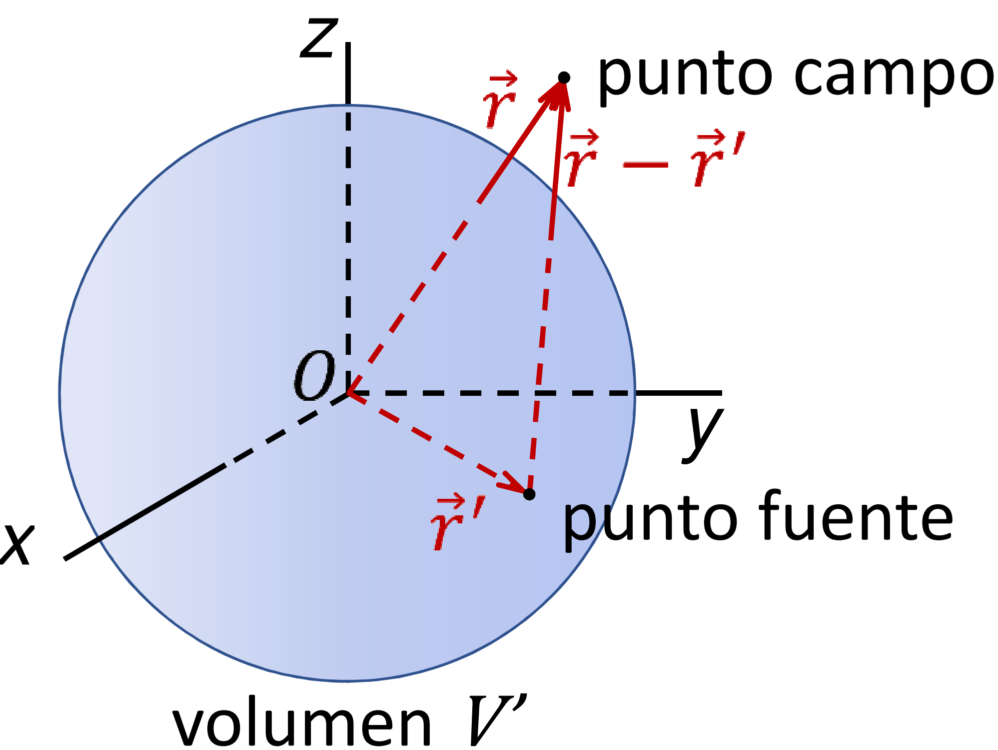

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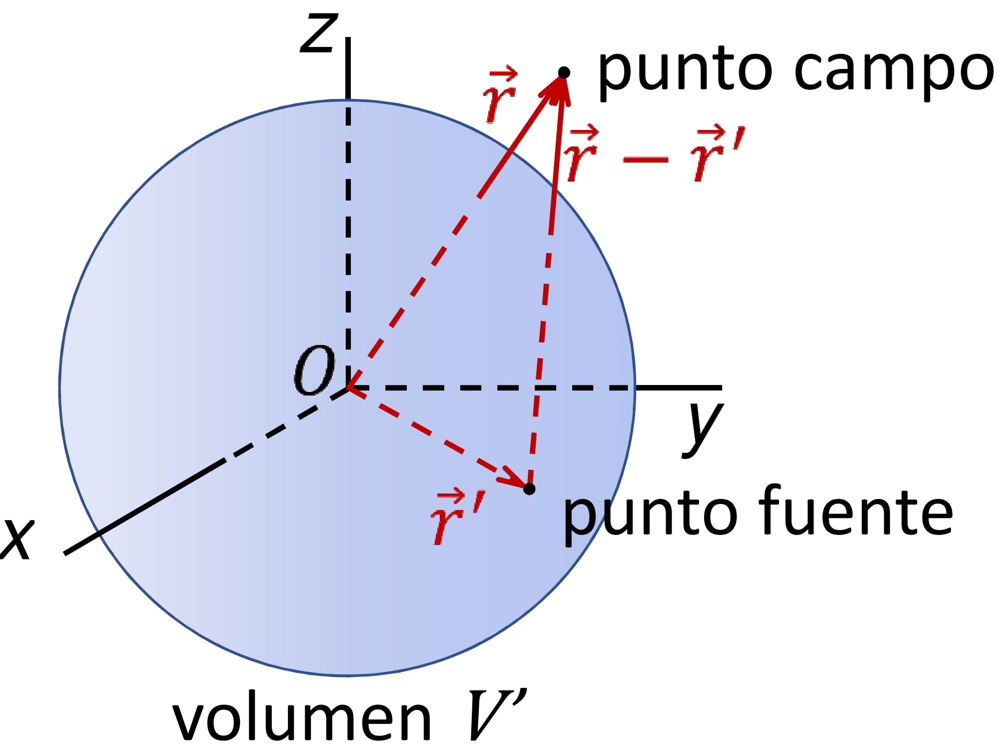

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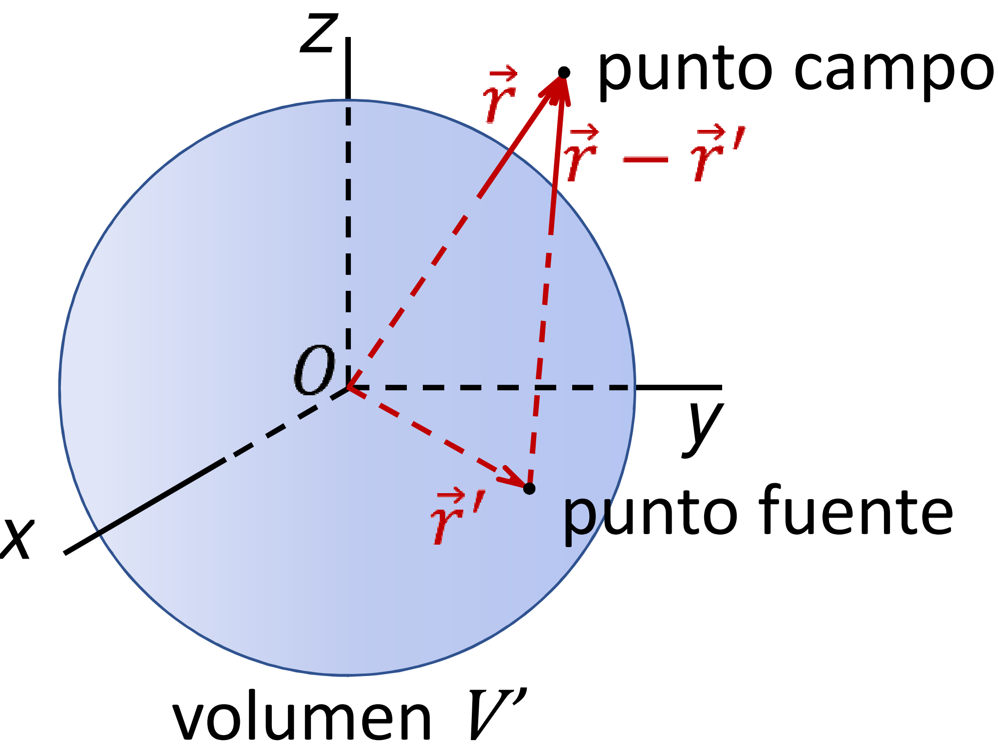

Pasos

1. $\mathrm{d}V'$
2. $\vec{r}\,'$
3. $\vec{r}$
4. $\vec{r}-\vec{r}\,'$
5. $|\vec{r}-\vec{r}\,'|$

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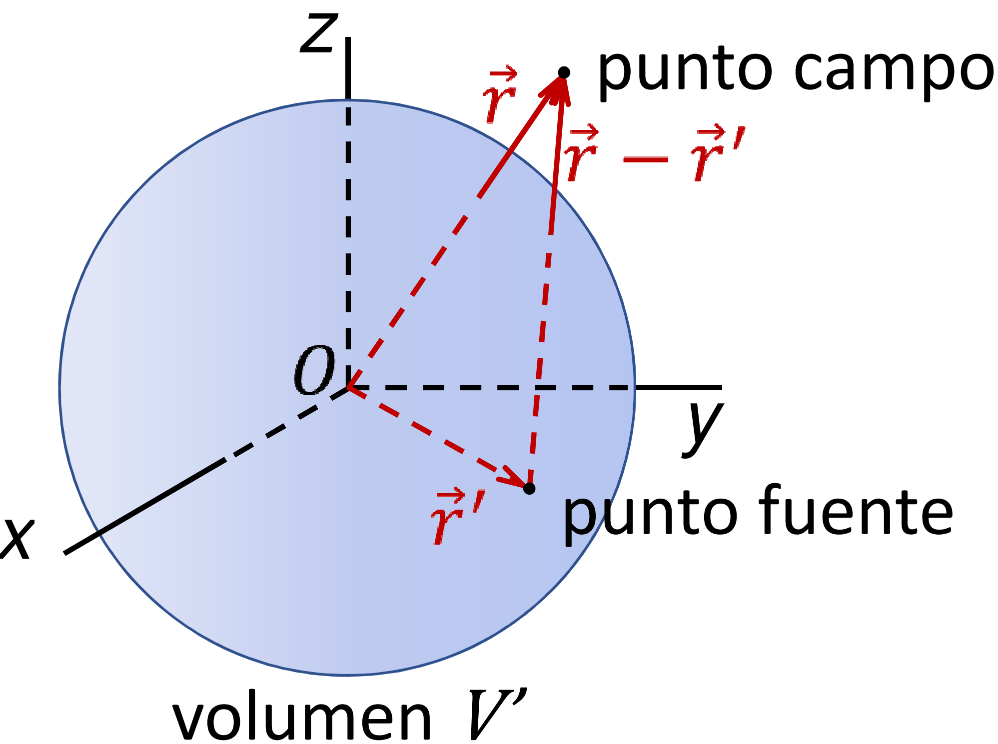

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coordenadas de integración esféricas: r', θ', φ'

os

$\mathrm{d}V'$

$$\mathrm{d}V' = r'^2 \,\mathrm{sen}\theta' \,\mathrm{d}r' \,\mathrm{d}\theta' \,\mathrm{d}\varphi'$$

$\vec{r}'$

$\vec{r}$

$\vec{r} - \vec{r}'$

$|\vec{r} - \vec{r}'|$

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$\mathrm{d}V'$

$\vec{r}'$

$\vec{r}$

$\vec{r}-\vec{r}'$

$|\vec{r}-\vec{r}'|$

$\vec{r}' = \vec{r}'(r', \theta', \varphi')$ vector con valor numérico y orientación en el espacio, **coordenadas** y **versores**

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$\vec{r}\,' = \vec{r}\,'(r', \theta', \varphi')$ vector con valor numérico y orientación en el espacio, **coordenadas** y **versores**

$$\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath} + z'\hat{\mathrm{k}}$$

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$$\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath} + z'\hat{\mathrm{k}}$$

$$\vec{r}\,' = r'\mathrm{sen}\theta'\cos\varphi'\,\hat{\imath} + r'\mathrm{sen}\theta'\mathrm{sen}\varphi'\,\hat{\jmath} + r'\cos\theta'\hat{\mathrm{k}}$$

mantenemos los versores cartesianos

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\mathrm{d}V'}{|\vec{r}-\vec{r}\,'|^3}$$

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coordenadas de integración esféricas: r', θ', φ'

os

$\mathrm{d}V'$

$\vec{r}\,'$

$\vec{r}$

$$\vec{r} = r\,\mathrm{sen}\theta\,\cos\varphi\,\hat{\imath} + r\,\mathrm{sen}\theta\,\mathrm{sen}\varphi\,\hat{\jmath} + r\cos\theta\,\hat{\mathrm{k}}$$

$\vec{r}-\vec{r}\,'$

$|\vec{r}-\vec{r}\,'|$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\mathrm{d}V'}{|\vec{r}-\vec{r}\,'|^3}$$

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$$\vec{r}\,' = r'\mathrm{sen}\theta'\cos\varphi'\,\hat{\imath} + r'\mathrm{sen}\theta'\mathrm{sen}\varphi'\,\hat{\jmath} + r'\cos\theta'\hat{\mathrm{k}}$$

$$\vec{r}-\vec{r}\,' = (r\,\mathrm{sen}\theta\,\cos\varphi - r'\mathrm{sen}\theta'\cos\varphi')\,\hat{\imath} + (r\,\mathrm{sen}\theta\,\mathrm{sen}\varphi - r'\mathrm{sen}\theta'\mathrm{sen}\varphi')\,\hat{\jmath} + (r\cos\theta - r'\cos\theta')\,\hat{\mathrm{k}}$$

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$$|\vec{r}-\vec{r}\,'| = \left[(r\,\mathrm{sen}\theta\,\cos\varphi - r'\mathrm{sen}\theta'\cos\varphi')^2 + (r\,\mathrm{sen}\theta\,\mathrm{sen}\varphi - r'\mathrm{sen}\theta'\mathrm{sen}\varphi')^2 + (r\cos\theta - r'\cos\theta')^2\right]^{1/2}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\mathrm{d}V'}{|\vec{r}-\vec{r}\,'|^3} \qquad \rho(\vec{r}\,') = \rho_0 \cos\varphi'$$

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rollando, agrupando, con $\mathrm{sen}^2\alpha + \cos^2\beta = 1$ y $\cos\alpha\cos\beta + \mathrm{sen}\alpha\,\mathrm{sen}\beta = \cos(\alpha -$

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rollando, agrupando, con $\mathrm{sen}^2\alpha + \cos^2\beta = 1$ y $\cos\alpha\cos\beta + \mathrm{sen}\alpha\,\mathrm{sen}\beta = \cos(\alpha$ -

$$\vec{r}-\vec{r}\,'| = [r^2 + r'^2 - 2rr'\mathrm{sen}\theta\,\mathrm{sen}\theta'\cos(\varphi-\varphi') - 2rr'\cos\theta\cos\theta']^1$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\mathrm{d}V'}{|\vec{r}-\vec{r}\,'|^3}$$

$$\rho(\vec{r}\,') = \rho_0 \cos\varphi'$$

os

$\mathrm{d}V'$ ✓

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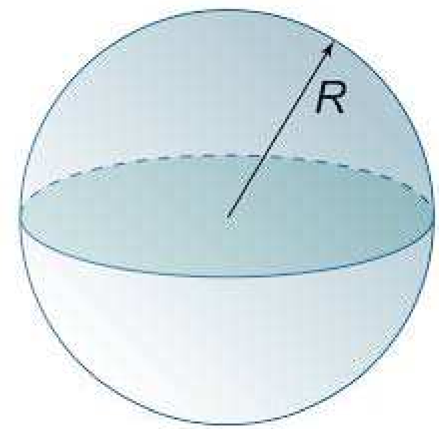

os

$\mathrm{d}V'$ ✓

$\vec{r}'$ ✓

$\vec{r}$ ✓

$\vec{r}-\vec{r}'$ ✓

$|\vec{r}-\vec{r}'|$ ✓

- $0 \le r' \le R$
- $0 \le \theta' \le \pi$
- $0 \le \varphi' \le 2\pi$

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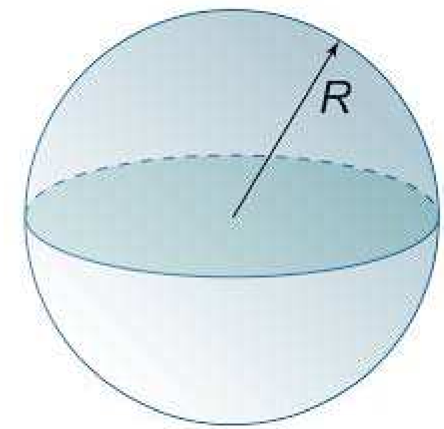

os

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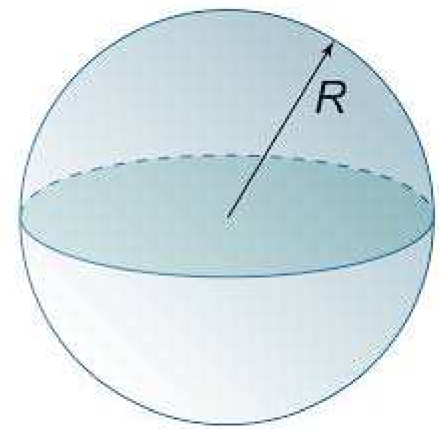

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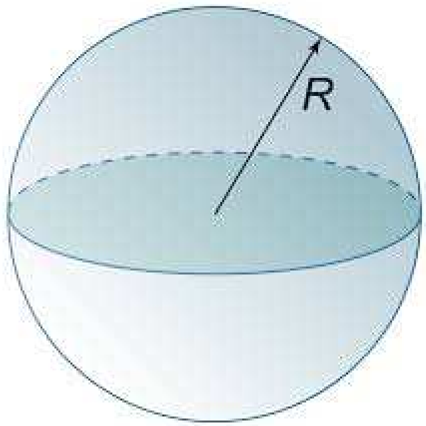

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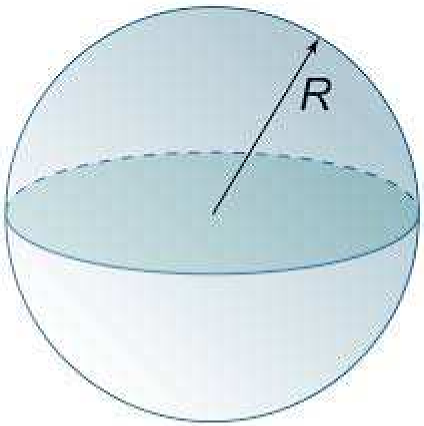
$$\vec{E}(\vec{r}) = E_x(r,\theta,\varphi)\,\hat{\imath} + E_y(r,\theta,\varphi)\,\hat{\jmath} + E_z(r,\theta,\varphi)$$



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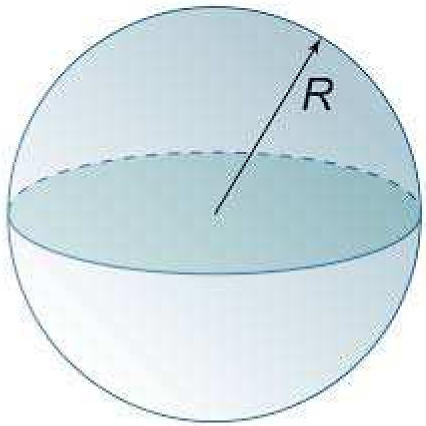

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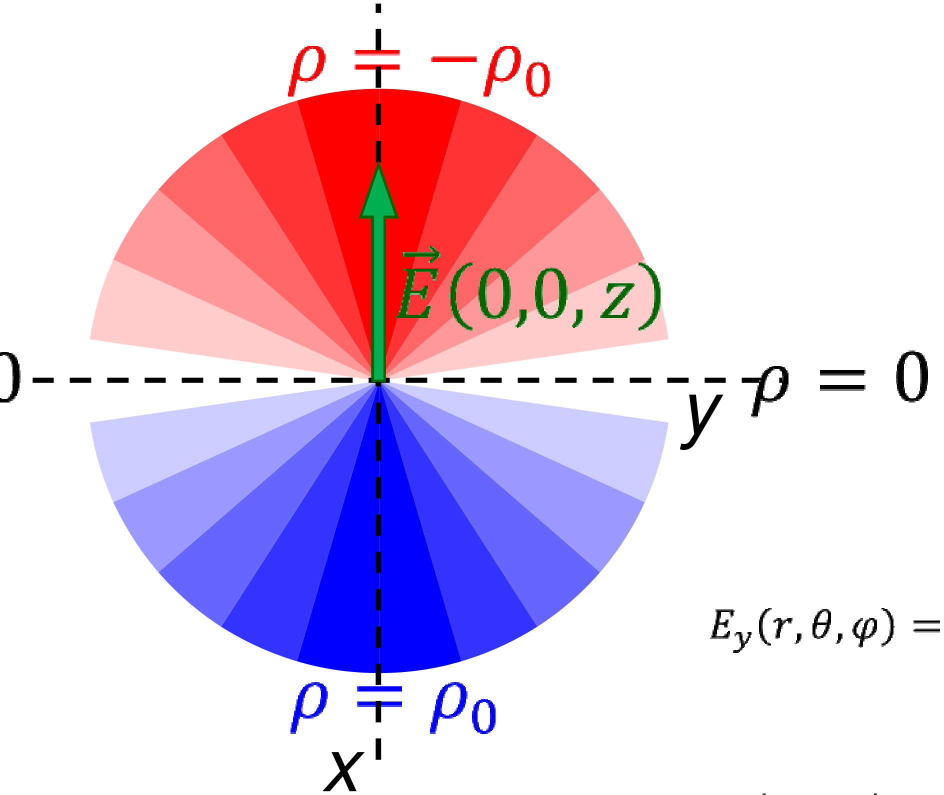

Por la forma de $\rho(\varphi)$, podemos deducir que en los puntos del eje z:

$$\vec{E}(0,0,z) = -|E|\hat{\imath}$$

Esto se puede obtener demostrando que las integrales

$$E_y(r,\theta,\varphi) = \frac{\rho_0}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{\cos\varphi'(r\,\text{sen}\theta\,\text{sen}\varphi - r'\text{sen}\theta'\text{sen}\varphi')\,r'^2\text{sen}\theta'\text{d}r'\text{c}}{[r^2 + r'^2 - 2rr'\text{sen}\theta\text{sen}\theta'\cos(\varphi-\varphi') - 2rr'\cos\theta\text{c}}$$

$$E_z(r,\theta,\varphi) = \frac{\rho_0}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{\cos\varphi'(r\cos\theta - r'\cos\theta')\,r'^2\text{sen}\theta'\text{d}r'\text{d}\theta'\text{d}\varphi'}{[r^2 + r'^2 - 2rr'\text{sen}\theta\text{sen}\theta'\cos(\varphi-\varphi') - 2rr'\cos\theta\text{c}}$$

dan resultado 0 cuando $\theta = 0$ o $\theta =$

Este problema resuelto es un material previo a las clases a distancia de Física II de la FIUBA. Por supuesto que en ese espacio podemos seguir hablando acerca de lo que acabo de contarles.
Gracias y saludos.

Rodolfo Aparicio